COMPUTATION OF CERTAIN DOMINATION NUMBERS OF SIGNED GRAPHS AND EXTENSION TOWARDS FUZZY GRAPHS

A SYNOPSIS

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1 Introduction

There are several reasons for the acceleration of interest in graph theory. It is worth to mention that there are applications of graph theory in various areas of physics, chemistry, communication science, computer technology, electrical and civil engineering, architecture, operational research, genetics, psychology, sociology and economics. Graph theory is also intimately related to many branches of mathematics, including group theory, matrix theory, numerical analysis, probability, topology, combinatorics etc. The fact is that graph theory serve as a mathematical model for any system involving a binary relation. Due to diagrammatic representation, graphs have an intuitive and aesthetic appeal. Although there are many results in this field of an elementary nature, there is also an abundance of problems with enough combinatorial subtlety to challenge the most sophisticated mathematician.

Signed graph theory forms one of the most vibrant areas of research due to its link with behavioral and social sciences as evident from the published literature; research-level journals like the Journal of Mathematical Sociology, Journal of Mathematical Psychology, Social Networks, Journal of Mathematical Chemistry etc., are only a few notable ones that can be mentioned in this context. In our excursion of research, we were mainly driven to carry out work in the area of signed graphs derived under domination. We were able to obtain some theoretical results which led us to the area of research in the theory of signed graphs with a hope to build necessary conceptual resources for applications.

Generally, a graph is an ordered pair $G = (V, E)$, where $V = V(G)$ and $E = E(G)$ are the vertex set and the edge set of the graph G respectively.

A signed graph $\Sigma = (G, \sigma)$ is a graph $G = (V, E)$ together with a signing function $\sigma : E \rightarrow \{+, -\}$ and edges which receive $+(-)$ signs are called positive (negative) edges of Σ, respectively.

In a pictorial representation of a signed graph Σ, its positive edges are shown as solid line segments and negative edges as dashed line segments.

To have a better understanding of its definition, we need to know the following notations.

For a vertex $v \in V(\Sigma)$, its $N^+(v) = \{u \in V : uv \in E(G)$ and $\sigma(uv) = +\}$ and its $N^-(v) = \{u \in V : uv \in E(G)$ and $\sigma(uv) = -\}$ are called the positive neighborhood and negative neighborhood of v respectively. Also $|N^+(v)|$ is called the positive degree of v and is denoted by $d^+(v)$ and $|N^-(v)|$ is called the negative degree of v and is denoted by $d^-(v)$. As in sighed graphs, we define
$\Delta^+ = max\ \{d^+(v) : v \in V\}$,
$\Delta^- = max\ \{d^-(v) : v \in V\}$,
$\delta^+ = min\ \{d^+(v) : v \in V\}$
$\delta^- = min\ \{d^-(v) : v \in V\}$
$E^+(\Sigma) = \{e \in E(\Sigma) : \sigma(e) = +\}$ and
$E^-(\Sigma) = \{e \in E(\Sigma) : \sigma(e) = -\}$.

A dominating set in a graph G is defined as follows.

Definition 1.0.1. *Let $G = (V; E)$ be a graph. A subset S of V is called a dominating set of G if each vertex not in S is adjacent to atleast one vertex in S. The cardinality of a smallest dominating set, denoted by $\gamma(G)$, is called the domination number of G and a dominating set having that cardinality is referred to as a $\gamma(G)$-set.*

A domination set in a signed graph Σ is defined as follows.

Definition 1.0.2. *Let $\Sigma = (V, E, \sigma)$ be a signed graph. A subset $D \subseteq V$ is a dominating set of Σ, if there exists a marking $\mu : V \to \{+, -\}$ such that every vertex $u \in V$ is either in D or whenever $u \in V \setminus D$, $N(u) \cap D \neq \emptyset$ and $\sigma(uv) = \mu(u)\mu(v)$ for every $v \in N(u) \cap D$. The minimum cardinality of a dominating set of Σ is called the domination number of Σ and it is denoted by $\gamma(\Sigma)$.*

We consider another way of defining the concept of domination in signed graphs which looks more natural and has applications in social science. Consider a social network represented by a signed graph Σ. If $S \subseteq V$ and $v \in V - S$, then a neighbor u of v in S has a positive or negative influence on v according as $\sigma(uv) = +$ or $\sigma(uv) = -$. Hence if $|N^+(v) \cap S| > |N^-(v) \cap S|$, then the overall influence of S on v can be taken as positive. Motivated by this observation, we introduce the following concept of domination in signed graphs.

Definition 1.0.3. *Let $\Sigma = (G, E, \sigma)$ be a signed graph. A subset S of V is called a dominating set of Σ if $|N^+(v) \cap S| > |N^-(v) \cap S|$ for all $v \in V - S$. The domination number of Σ, denoted by $\gamma_s(\Sigma)$, is the minimum cardinality of a dominating set of Σ. Also a dominating set S of Σ with $|S|$=γ_s is called a γ_s- set of Σ.*

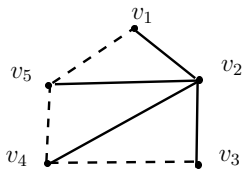

Figure 1.1: Signed Graph.

From the Figure 1.1, the set $\{v_2\}$ is a dominating set of Σ, $\gamma_s(\Sigma) = 1$.

2 Literature Review

Harary [1] introduced the signed graph. In 1984, Sampathkumar [2] characterized the balance of signed graph. In 2013, Acharya [3] introduced the *domination number* in signed graph. After Germina and Ashraf [4, 5] gave characterization for *open domination and double domination* in signed graphs. In 2015, Walikar et al. [6] introduced the

concept of *signed domination* for signed graphs. This has motivated us to investigate the possibility of computing the various domination numbers in the signed graphs.

Graph theory is an indispensable part of mathematics. The underlying algebraic structure of a graph makes it a tool to model real-life challenges in many areas of sciences [7]. The advent of the fuzzy graph, as a generalization of a graph provides more information to real-life challenges. Zadeh [8] introduced the fuzzy relations. Kaufman's [9] definition of a fuzzy graph enabled Rosenfeld [10]. Yeh and Bang [11] introduced the algebraic structures of fuzzy graph theory.

After that, domination in fuzzy incidence graphs based on valid edges [12], the energy of double dominating bipolar fuzzy graphs [13], complementary nil domination in fuzzy graphs [14], strong domination in fuzzy graphs [15], connected domination in fuzzy graphs using strong edges [16], double vertex-edge domination in graphs [17], have become popular. A $(1, 2)$-vertex domination in fuzzy graphs [18] and relation between efficient domination number and the chromatic number of a fuzzy graph [19] have attracted attention of the researchers. From this motivation, we actually venture into such study we thought it fit to probe the apex and bottom vertices of fuzzy graphs and analysis some of its properties and then try to extend to the more generalized fuzzy signed graphs. Nagoor Gani and Ratha [20] introduced fuzzy regular graphs, total degree and totally regular fuzzy graphs. Ramakrishnan and Lakshmi [21, 22] introduced depth of μ, height of μ. With this in mind we were successful in deriving some new interesting properties concerning apex and bottom vertices of a fuzzy graph.

3 Objectives

The main objectives of this research are

1. To introduce and initiate the concept of domination in signed graphs.
2. To investigate the variations of domination parameters such as connected domination number and independent domination number in signed graphs.
3. To study the nature of the parameter by vertex deletion in signed graphs.
4. To analyse the the apex and bottom vertices of fuzzy graphs.

4 Organization of Thesis

The thesis comprises of six chapters.

Chapter 1. Introduction

Chapter 2. Domination in Signed Graphs

Chapter 3. Connected Domination in a Signed Graph and its Complement

Chapter 4. Independent Domination in Signed Graphs

Chapter 5. Domination Changing and Unchanging Signed Graphs Upon the Vertex

Chapter 6. Apex of fuzzy vertices and bottom of fuzzy vertices in a fuzzy graph

4.1 Introduction

In **Chapter 1**, we give some basic definitions and results which motivated our research. By a graph $G = (V, E)$, we mean a finite undirected graph with neither loops nor multiple edges. For graph theoretic terminology we refer to Chartrand and Lesniak [23].

4.2 Domination in signed graphs

In **Chapter 2**, we continue the study of this parameter. We determine the domination number of several classes of signed graphs such as signed path and signed cycle graph. We also give an upper bound and lower bound for $\gamma_s(\Sigma)$ and investigate the structure of signed graphs with domination number. The following are some of the significant results.

Lemma 4.2.1. *Let $\Sigma = (V, E, \sigma)$ be a signed graph. Let $v \in V$. If $d^-(v) \geq n - \lfloor \frac{\gamma_s}{2} \rfloor$, then v lies in every γ_s-set S of Σ.*

Corollary 4.2.2. *Let $\Sigma = (V, E, \sigma)$ be a signed graph. If $d^-(v) \geq n - \lfloor \frac{\gamma_s}{2} \rfloor$ for all $v \in V$, then $\gamma_s(\Sigma) = n$.*

Remark 4.2.3. *If $deg(v) = 1$ and if the edge uv is a negative edge, then v lies in every dominating set of Σ.*

Theorem 4.2.4. *Let Σ be a signed graph whose underlying graph is $K_{1,n}$. Then $\gamma_s(\Sigma) = k$ if and only if Σ contains exactly $k - 1$ negative edges.*

Corollary 4.2.5. *Let k and n be positive integers with $1 \leq k \leq n$. Then there exists a signed graph Σ such that $\gamma_s(\Sigma) = k$.*

The following theorem gives the maximum number of negative edges of a signed graph when $\gamma_s(\Sigma) \geq 1$.

Theorem 4.2.6. *If a signed graph Σ with order n, $n \geq 2$ and $\gamma_s(\Sigma) \geq 1$ then number of negative edges of Σ is denoted by m^- and*

$$m^- \leq \frac{n(n-2) + \gamma_s(\gamma_s - n + 1)}{2}.$$

Remark 4.2.7. *If any signed tree Σ of order n, $n \geq 2$ with k positive leaves then $\gamma_s(\Sigma) \leq n - k$.*

Theorem 4.2.8. *Let Σ be a signed graph. Then $\gamma_s \geq \frac{n}{1+\Delta^+}$.*

Theorem 4.2.9. *Let Σ be a signed graph. Then $\gamma_s(\Sigma) \leq 2(n-1-\delta)$ where $\delta = min\{\delta^+, \delta^-\}$.*

Remark 4.2.10. *For any signed graph Σ, $\gamma_s(\Sigma) = 1$ if and only if $\Delta^+(\Sigma) = n-1$.*

Definition 4.2.11. *Let Σ be a signed graph and let S be a proper nonempty subset of V. Let $[S, V-S]$ denote the set of all edges of Σ with one end in S and other end in $V-S$ where the edges have the same sign as in Σ. We denote by Σ_S the spanning subgraph of Σ with edge set $[S, V-S]$. Σ_S has same vertex set as Σ.*

Theorem 4.2.12. *Let Σ be a signed graph and let $\gamma_s(\Sigma) = k$. Let S be a γ_s-set of a signed graph Σ. Then $0 \leq d^-_{\Sigma_s}(v) < \frac{\gamma_s}{2}$ for all $v \in V-S$.*

We characterized the domination number in Σ.

Theorem 4.2.13. *Let Σ be a signed graph of order n. Then $\gamma_s(\Sigma) = 2$ if and only if the following conditions hold:*
(i) $\Delta^+(\Sigma) < n-1$.
(ii) There exists a subset $S \subseteq V$ such that $|S| = 2$ and Σ_S is all positive.

Corollary 4.2.14. *Let Σ be a signed graph whose underlying graph is K_n. Then $\gamma_s(\Sigma) = 2$ if and only if $\Delta^+ < n-1$ and the subgraph H of Σ induced by the set of positive edges is a supergraph of $K_{2,n-2}$.*

Theorem 4.2.15. *Let $\Sigma = (V, E, \sigma)$ be a signed graph of order n. Then $\gamma_s(\Sigma) = 3$ if and only if the following conditions hold:*
(i) For any subset $S \subseteq V(\Sigma)$ with $|S| \leq 2$, the negative subgraph of Σ_S is nonempty.
(ii) There exists a subset $S \subseteq V$ such that $|S| = 3$ satisfying the following conditions:
(a) The negative subgraph of Σ_S has at most three components and each of these components is either an isolated vertex or a star.
(b) $d^+_{\Sigma_s}(v) > d^-_{\Sigma_s}(v)$ for all $v \in V-S$.

Theorem 4.2.16. *Let $\Sigma = (V, E, \sigma)$ be a signed graph of order n. Then $\gamma_s(\Sigma) = 4$ if and only if the following conditions hold*
(i) For every subset $S \subseteq V(\Sigma)$ with $|S| \leq 2$, the negative subgraph of Σ_S is nonempty.
(ii) For every subset $S \subseteq V$ such that $|S| = 3$, there exists a vertex $v \in V-S$ with $d^-_{\Sigma_s}(v) \geq d^+_{\Sigma_s}(v)$.
(iii) There exists a subset $S \subseteq V$ such that $|S| = 4$ satisfying the following conditions:
(a) The negative subgraph of Σ_S has at most four components, and each of these components is either an isolated vertex or a star.
(b) $d^+_{\Sigma_s}(v) > d^-_{\Sigma_s}(v)$ for all $v \in V-S$.

We calculated exact values of domination numbers for signed paths, signed cycles.

Theorem 4.2.17. *Let Σ be a signed graph obtained from $P_n = (v_1, v_2, ..., v_n)$ and let $n \geq 3$ by assigning positive and negative sign to the edges alternatively. Then*

$$\gamma_s(\Sigma) = \begin{cases} \lceil \frac{n}{2} \rceil & : n \text{ is odd} \\ \frac{n}{2} & \text{if } n \text{ is even and both } v_1v_2 \text{ and } v_{n-1}v_n \text{ are in } E^+ \\ \frac{n}{2}+2 & : n \equiv 0(mod\,4)\ v_1v_2 \text{ and } v_{n-1}v_n \text{ are in } E^- \\ \frac{n}{2}+1 & : n \equiv 2(mod\,4) \text{ and } v_1v_2 \text{ and } v_{n-1}v_n \text{ are in } E^- \end{cases}$$

Theorem 4.2.18. *Let Σ be a signed graph obtained from the cycle $C_n = (v_1, v_2, ..., v_n, v_1)$ by assigning positive and negative sign to the edges alternatively. Then*

$$\gamma_s(\Sigma) = \begin{cases} \frac{n}{2} & : n \equiv 0(mod\,4) \\ \lfloor \frac{n}{2} \rfloor + 1 & : n \equiv 1 \text{ or } 2(mod\,4) \\ \lfloor \frac{n}{2} \rfloor & : n \equiv 3(mod\,4) \text{ if } d^+(v_1) = 2 \\ \lceil \frac{n}{2} \rceil + 1 & : n \equiv 3(mod\,4) \text{ if } d^-(v_1) = 2. \end{cases}$$

Problem 4.2.19. *Characterize the signed graphs for which $\gamma_s(\Sigma) = n$.*

Problem 4.2.20. *Let Σ be a signed graph. Let Σ_S be a spanning subgraph of Σ. Characterize the properties of Σ_S.*

4.3 Connected Domination in a Signed graph and its Complement

Several conditional dominating sets have been formulated and investigated by imposing a condition on the subgraph induced by a dominating set. Sampathkumar and Walikar [24] introduced the concept of connected domination in graphs. A dominating set S of G is called a connected dominating set if the induced subgraph $G[S]$ is a connected subgraph of G. The minimum cardinality of a connected dominating set of G is called the connected domination number of G and is denoted by $\gamma_c(G)$. Motivated by this concept we initiated the concept of connected dominating set and connected domination number of a signed graph.

In **Chapter 3**, we continue the study of this parameter. We determine the connected domination number of several classes of signed graphs, investigate the structure of signed graphs with connected domination number and Nordhaus - Gaddum type results for connected domination number are established.

A connected dominating set of Σ is defined as follows.

Definition 4.3.1. *A* dominating set *D of Σ is called a* connected dominating set*, if the subgraph of Σ induced by D, denoted by $\Sigma[D]$, is connected. The* connected domination number *of Σ, denoted by $\gamma_{sc}(\Sigma)$, is the minimum cardinality of a connected dominating set of Σ. A* connected dominating set *of Σ of cardinality $\gamma_{sc}(\Sigma)$ is called a $\gamma_{sc}(\Sigma)$-set.*

We present some basic properties of connected dominating sets.

Observation 4.3.2. *1. $\gamma_{sc}(\Sigma) \geq \gamma_s(\Sigma) \geq \gamma(G)$, G is an unsigned graph of Σ.*

2. *For any connected signed graph Σ, $\gamma_{sc}(\Sigma) = 1$ if and only if there exists a vertex v such that $deg^+(v) = n - 1$. Thus, $\gamma_{sc}(\Sigma) = 1$ if and only if $\Sigma = \Sigma^* + K_1$ for some connected signed graph Σ^* such that each vertex $v \in V(\Sigma^*)$ is adjacent to K_1 by a positive edge.*

3. *For every connected signed graph Σ on $n \geq 2$ vertices $\gamma_{sc}(\Sigma) \leq n$ with equality if and only if $\Sigma = K_2^-$.*

Lemma 4.3.3. *Let Σ be a signed graph. Then $\gamma_{sc} \geq \frac{n}{1+\Delta^+}$.*

Lemma 4.3.4. *Let S be a γ_{sc}-set of Σ of order $n \geq 3$. Then*
(i) Every cut vertex of Σ belongs to S.
(ii) Every support vertex of Σ belongs to$S$.
(iii) Every negative pendant vertex of Σ belongs to S.

Theorem 4.3.5. *Let Σ be a signed tree of order n*

$$\gamma_{sc}(\Sigma) = n - |E_p^+|$$

where E_p^+ is the number of positive pendant vertices in Σ.

Proposition 4.3.6. *For any connected signed graph Σ has order n and size m with a maximum positive degree Δ^+*

$$\frac{n}{1+\Delta^+} \leq \gamma_{sc} \leq 2m - n + 2.$$

Further, the upper bound is attained if and only if Σ is a fully-negative signed tree.

Theorem 4.3.7. *For any signed graph Σ,*

$$diam(\Sigma) - 1 \leq \gamma_{sc}(\Sigma).$$

Theorem 4.3.8. *For any signed graph Σ, $\gamma_{sc}(\Sigma) \leq n - \Delta^+$.*

We discuss the existence of signed graphs with $\gamma_s(\Sigma) = a$ and $\gamma_{sc}(\Sigma) = b$ where a and b are two positive integers $a \leq b \leq 2a$.

Theorem 4.3.9. *Let a and b be two positive integers of $a \leq b \leq 2a$. Then there is a signed graph Σ with $\gamma_s(\Sigma) = a$ and $\gamma_{sc}(\Sigma) = b$.*

We calculated exact values of connected domination numbers for signed wheels, splitting signed graphs, book signed graphs,ladder signed graphs.

Theorem 4.3.10. *Let $W_{n+1} = K_1 + C_n$ be a signed wheel graph with $n \geq 3$ such that the cycle edges are assigned the sign $+$ and $-$ alternatively starts from $+$ and the spokes may be assigned by any sign. Let Σ be the signed graph obtained from the signed wheel graph W_{n+1} by subdividing each edge of the cycle $C_n = (v_1, v_2, ..., v_n, v_1)$ and having the sign of the subdivision as in the sign of W_{n+1}. Then*

$$\gamma_{sc}(\Sigma) = \begin{cases} 3\lfloor \frac{n}{2} \rfloor + 1 & : n \text{ is even,} \\ 3\lfloor \frac{n}{2} \rfloor + 1 & : n \text{ is odd and } \Delta^+(C_n) = 3 \\ 3\lfloor \frac{n}{2} \rfloor + 2 & : \text{otherwise} \end{cases}$$

Definition 4.3.11. *Let Σ be a signed graph. For each vertex $v \in \Sigma$, take a duplicated vertex v'. Join v' to all vertices of Σ adjacent to v and the sign of the edge uv is the same as the sign of uv' for each $u \in N(v)$. The resulting signed graph $S(\Sigma)$ is called the splitting signed graph of Σ.*

Observation 4.3.12. *Every $\gamma_s(\Sigma)$-set of $S(\Sigma)$ need not be a $\gamma_{sc}(\Sigma)$-set of $S(\Sigma)$.*

Theorem 4.3.13. *Let $S(\Sigma)$ be a splitting signed graph whose underlying signed graph is Σ. Then $\gamma_{sc}(S(\Sigma)) = 2\gamma_{sc}(\Sigma)$.*

Theorem 4.3.14. *Let $\Sigma^* = K_{1,n-1} \square K_2$ be a signed graph. Let k be the number of positive pendant vertices and l be the number of negative pendant vertices in $K_{1,n-1}$ such that $k + l = n - 1$. Then*

$$\gamma_{sc}(\Sigma^*) = \begin{cases} n & : \text{if } l \geq k \\ 2\gamma_{sc}(\Sigma) & : \text{otherwise} \end{cases}$$

Theorem 4.3.15. *Let $\Sigma^* = P_n \square P_2$ be a signed graph. Then*

$$\gamma_{sc}(\Sigma^*) = \begin{cases} n & : P_2 \text{ is fully positive} \\ 2\gamma_{sc}(\Sigma) & : \text{otherwise} \end{cases}$$

Let Σ_S be the spanning subgraph of Σ, $S \subset V(\Sigma)$. The edge set $[S, V - S]$ of Σ_S is a set of all edges having one end in S and the other end in $V - S$ and the sign of the edges of Σ_S is the same sign as that of Σ. Then $E(\Sigma) = E(S) \cup E(V - S) \cup E(\Sigma_S)$. We may write to Σ as $S \overline{\cup} (V - S)$.

Theorem 4.3.16. *Let Σ be a signed graph and let $\gamma_{sc}(\Sigma) = k$. Let S be a γ_{sc}-set of a signed graph Σ. Then $0 \leq d^-_{\Sigma_{sc}}(v) < \frac{\gamma_{sc}}{2}$ for all $v \in V - S$.*

We characterized the connected domination number of Σ.

Theorem 4.3.17. *Let Σ be a signed graph of order n. Then $\gamma_{sc}(\Sigma) = 2$ if and only if the following conditions hold*
(i) $\Delta^+(\Sigma) < n - 1$.
(ii) There exists a subset $S \subseteq V$ such that $|S| = 2$ satisfying the following conditions:
(a) S is connected
(b) $E^-(\Sigma_S) = \emptyset$.

Theorem 4.3.18. *For any Σ, $\gamma_{sc}(\Sigma) = 3$ if and only if*

(i) For any subset S of Σ with $|S| \leq 2$, and $E^-(\Sigma_s) \neq \emptyset$.
(ii) There exists a subset $S \subseteq V$ such that $|S| = 3$ satisfying the following conditions:
(a) S is a connected set
(b) $d^+_{\Sigma_S}(v) > d^-_{\Sigma_S}(v)$ for all $v \in G = V - S$.

We found some signed graphs for which some of the parameters are equal.

Theorem 4.3.19. *For any signed tree Σ, $\gamma_s = \gamma_{sc}$ if and only if each branch vertex $v \in \Sigma$ is a support vertex.*

Theorem 4.3.20. *Let Σ be a signed unicyclic graph with cycle $C = v_1, v_2, ..v_n, v_1$, $n \geq 3$ and let P be the set of vertices of positive degree two in C. Then $\gamma_s(\Sigma) = \gamma_{sc}(\Sigma)$ if and only if the following conditions hold:*
(i) Each vertex v of degree at least two in $V - V[P]$ is a support vertex or has at least one negative neighbor.
(ii)$< P >$ is connected and $|P| \leq 2$.
(iii) If $< P >= K_1 or K_2$, at least one vertex in $N(P)$ is a support vertex or has at least one negative neighbor.

Nordhaus and Gaddum provided best possible bounds on the sum of the chromatic numbers of a graph and its complement in [25]. We studied signed graph relationships involving connected domination parameters in a signed graph Σ and its complement. We define the complement of a signed graph Σ.

Definition 4.3.21. *For a signed graph Σ, the complement of Σ is denoted by Σ^c_α is defined as follows. Σ^c_α has the same vertex set as Σ and the edge set $E' = \{uv : uv$ is a positive edge of Σ or $uv \notin E\}$ and $\sigma' : E' \to \{+, -\}$ is defined as*

$$\sigma'(uv) = \begin{cases} - & \text{if } \sigma(uv) = + \\ + & \text{if } uv \notin E(\Sigma) \end{cases}$$

Figure 4.2: Signed Graphs.

Example 4.3.22. *In the Figure 4.2, Σ is a signed graph and Σ^c_α is a complement of Σ.*

Let $V(\Sigma) = V(\Sigma^c_\alpha) = \{v_1, v_2, v_3, v_4\}$.

Here, $\sigma(v_1v_2) = + \Rightarrow \sigma'(v_1v_2) = -$,

$\sigma(v_2v_3) = - \Rightarrow \sigma'(v_2v_3) \notin E'$,

$\sigma(v_3v_4) = + \Rightarrow \sigma'(v_3v_4) = -$,

$\sigma(v_4v_2) = - \Rightarrow \sigma'(v_4v_2) = \notin E'$,

$\sigma(v_4v_1) \notin E(\Sigma) \Rightarrow \sigma'(v_4v_1) = +$, *and so on.*

Here, the set $\{v_2, v_3, v_4\}$ is a connected dominating set of Σ, $\gamma_{sc}(\Sigma) = 3$ and the set $\{v_1, v_2\}$ is a connected dominating set of Σ^c_α, $\gamma_{sc}(\Sigma^c_\alpha) = 2$.

Following are some of the significant results.

Theorem 4.3.23. *For Σ, $3 \leq \gamma_{sc}(\Sigma) + \gamma_{sc}(\Sigma^c_\alpha)$. More over, $\gamma_{sc}(\Sigma) + \gamma_{sc}(\Sigma^c_\alpha) = 3$ if and only if $\{\Sigma, \Sigma^c_\alpha\} = (K_2^+, K_2^-)$.*

Theorem 4.3.24. *For any connected signed graph Σ of order $n \geq 3$, $4 \leq \gamma_{sc}(\Sigma) + \gamma_{sc}(\Sigma_\alpha^c)$. More over, $\gamma_{sc}(\Sigma) + \gamma_{sc}(\Sigma_\alpha^c) = 4$ if and only if $\Sigma = K_3$.*

Theorem 4.3.25. *Let Σ be a connected signed graph of order $n \geq 3$ such that both Σ and Σ_α^c are connected. Then*

$$\gamma_{sc}(\Sigma) + \gamma_{sc}(\Sigma_\alpha^c) \leq n(n-3) + 2.$$

The bound is attained if and only if $\Sigma = P_4^-$.

Definition 4.3.26. *Let $n \geq 4$ be an integer. The signed graph L_n consists of a signed clique Σ_{n-2} plus two pendent edges ux and vy attached at two different vertices u and v of Σ_{n-2}.*

Definition 4.3.27. *The signed graph H_n consists of a complete signed split graph which is the join of an independent set of order $n-4$ and a signed clique of order 2 vertices x, y plus two pendent edges xt, yz where t, z are two independent vertices.*

Observation 4.3.28. *H_n^+ is a complement of L_n^-.*

Proposition 4.3.29. *For $n \geq 5$, $\gamma_{sc}(L_n^-) = n$ and $\gamma_{sc}(H_n^+) = 2$.*

Lemma 4.3.30. *If Σ and Σ_α^c are connected on n vertices and $diam(\Sigma) \geq 4$, then $\gamma_{sc}(\Sigma) + \gamma_{sc}(\Sigma_\alpha^c) \leq n+3$.*

Lemma 4.3.31. *If Σ and Σ_α^c are connected on n vertices and $diam(\Sigma) = 3$, then $\gamma_{sc}(\Sigma) + \gamma_{sc}(\Sigma_\alpha^c) \leq n+3$.*

Theorem 4.3.32. *Let Σ and Σ_α^c be connected on n vertices with $diam(\Sigma) = diam(\Sigma_\alpha^c) = 3$. Then $\gamma_{sc}(\Sigma) + \gamma_{sc}(\Sigma_\alpha^c) \leq n+2$. Equality occurs if and only if $n \geq 4$ and Σ is the signed graph L_n^-.*

Problem 4.3.33. *Characterize the signed graphs for which $\gamma_{sc}(\Sigma) = n$.*

Problem 4.3.34. *Characterize the class of signed graphs Σ for which $\gamma_s(\Sigma) = \gamma_{sc}(\Sigma)$.*

4.4 Independent domination in signed graphs

The concept of independent dominating set was introduced by Berge [26] and Ore [27]. A subset S of vertices is independent if $G[S]$ has no edge. A set $S \subseteq V(G)$ is an independent dominating set if S is an independent and dominating set. The minimum cardinality of such a set is the independent domination number $i(G)$. The parameter $i(G)$ was introduced by Cockayne and Hedetniemi [28]. This situation is a motivation for the following definitions.

Definition 4.4.1. *Let $\Sigma = (V, E, \sigma)$ be a signed graph. A subset $S \subseteq V$ is said to be fully negative or independent if each vertex $v \in S$ is either an isolated vertex or a vertex adjacent to the other vertices of $S - \{v\}$ by negative edge. That is every edge in $< S >$ is a negative edge.*

Definition 4.4.2. *Let $\Sigma = (V, E, \sigma)$ be a signed graph. A dominating set $S \subseteq V$ is an independent dominating set if $< S >$ is a fully negative. The minimum cardinality of such a set is the independent domination number $i_s(\Sigma)$.*

Following are some of the basic results.

Observation 4.4.3. *For a signed graph $\Sigma = (V, E, \sigma)$, $\gamma_s(\Sigma) \leq i_s(\Sigma)$.*

Observation 4.4.4. *Let Σ be a signed graph. Then $i_s \geq \frac{n}{1+\Delta^+}$.*

Theorem 4.4.5. *For a signed graph Σ, $1 \leq i_s(\Sigma) \leq n$. Further*

1. *$i_s(\Sigma) = 1$ if and only if $\Delta^+(\Sigma) = n - 1$.*
2. *$i_s(\Sigma) = n$ if and only if Σ is a negative signed graph.*

Theorem 4.4.6. *Let Σ' be a signed graph obtained from a signed graph Σ on n vertices by attaching a positive leaf at each vertex. Then $\gamma_s(\Sigma) + i_s(\Sigma) = 2n$.*

Theorem 4.4.7. *Let Σ be a signed graph and let S be an i_s-set such that $< V - S >$ is connected. Then $diam(\Sigma) \leq n - i_s(\Sigma) + 1$.*

Theorem 4.4.8. *Let Σ be a signed star graph. Then $\gamma_s(\Sigma) = i_s(\Sigma)$.*

We discuss the existence of signed graphs with $i(G) = a$ and $i_s(\Sigma) = b$ where a and b are two positive integers $a \leq b$.

Theorem 4.4.9. *Let a and b be two positive integer with $a \leq b$. Then there exists a signed graph Σ and its underlying graph G such that $i(G) = a$ and $i_s(\Sigma) = b$, $a \geq 1$.*

We determine the value of $i_s(\Sigma)$ for two families of signed graphs.

Theorem 4.4.10. *In a signed path Σ,*

1. *$i_s(\Sigma) = \lfloor \frac{n}{2} \rfloor + 1$ if $n \neq 0(mod 4)$*
2. *if the edges are alternatively signed by $+$ and $-$, $i_s(\Sigma) = \frac{n}{2}$ for $n \equiv 0(mod 4)$*

Remark 4.4.11. *In a signed path Σ, if the edges of Σ are signed by negative and positive,then independent dominating set does not exist.*

Theorem 4.4.12. *Let Σ be a signed graph. An independent set D of Σ is maximal independent if and only if it is independent and dominating of Σ.*

Problem 4.4.13. *Characterization for $i_s(\Sigma) = k$, $1 \leq k \leq n$.*

Problem 4.4.14. *Characterize signed graphs for which $\gamma_s(\Sigma) = i_s(\Sigma)$.*

4.5 Domination Changing and Unchanging Signed Graphs Upon the Vertex Removal

Harary [29] was the first to propose the terms changing and unchanging. The critical vertex of domination in a graph G is the vertex with a reduction in the domination number.

In **Chapter 5,** we studied those signed graphs, in which when any vertex is removed, the dominion number increases or decreases. Also we studied these signed graphs, in which when every vertex is removed the domination number remains unchanged.

Let $\Sigma - v$ represent the signed graph formed by removing the vertex v from Σ.

A signed graph is shown in the following classes.

$$\gamma_s(\Sigma - v) \neq \gamma_s(\Sigma) \text{ for all } v \in V(\Sigma)$$

$$\gamma_s(\Sigma - v) = \gamma_s(\Sigma) \text{ for all } v \in V(\Sigma)$$

Individual studies of these two classes have been conducted in the literature.

Definition 4.5.1. *If $\gamma_s(\Sigma - v) \neq \gamma_s(\Sigma)$ for all $v \in V(\Sigma)$, then the signed graph is called a* vertex critical *signed graph.*

Definition 4.5.2. *If $\gamma_s(\Sigma - v) = \gamma_s(\Sigma)$ for all $v \in V(\Sigma)$, then the signed graph is called a* stable *signed graph.*

Definition 4.5.3. *Let Σ be a signed graph, we define a partition of $V(\Sigma) = V^0(\Sigma) \cup V^+(\Sigma) \cup V^-(\Sigma)$ of its vertex set, where*

$$V^0(\Sigma) = \{v \in V(\Sigma) : \gamma_s(\Sigma - v) = \gamma_s(\Sigma)\}$$
$$V^+(\Sigma) = \{v \in V(\Sigma) : \gamma_s(\Sigma - v) > \gamma_s(\Sigma)\}$$
$$V^-(\Sigma) = \{v \in V(\Sigma) : \gamma_s(\Sigma - v) < \gamma_s(\Sigma)\}$$

We simply write V^0, V^+, and V^-.

We proved some useful properties of V^+ and V^-.

Proposition 4.5.4. *(a) If I denotes the collection of all isolated vertices of Σ, then $I \subseteq V^-$.*
(b) $|V^+| \leq \gamma_s(\Sigma)$.

Theorem 4.5.5. *A vertex $v \in V^-(\Sigma)$ if and only if $N^-(v) \cap S \neq \emptyset$ for some γ_s-set S containing v.*

Theorem 4.5.6. *A vertex $v \in V^-(\Sigma)$ if and only if there exists some $\gamma_s(\Sigma)$-set S and a vertex $u \in S$ such that $v \notin S$ and $S \cap N^+(v) = \{u\}$.*

Proposition 4.5.7. *If $\gamma_s(\Sigma) = 1$, then $V^- = \emptyset$.*

Proposition 4.5.8. *Every vertex $v \in V^+$ belong to each γ_s-set of Σ.*

Theorem 4.5.9. *A vertex v belongs $V^+(\Sigma)$ if and only if*
(a) v is in every γ_s-set of Σ.
(b) There is no subset $S \subseteq V(\Sigma) - N[v]$ with cardinality $\gamma_s(\Sigma)$ that dominates $\Sigma - v$.

Proposition 4.5.10. *If $A_s(v)$ has at least two vertices, then $\gamma_s(\Sigma - v) > \gamma_s(\Sigma)$.*

Observation 4.5.11. *If $v \in V^+$, then v has at least one negative neighbor in S.*

Theorem 4.5.12. *Let Σ be a signed graph of order n. Then for every vertex $v \in V(\Sigma)$,*

$$\gamma_s(\Sigma) - 2 \leq \gamma_s(\Sigma - v) \leq \gamma_s(\Sigma) - 1 + deg(v) + k,\ k < n.$$

We defined the $i-$ stability and $d-$ stability of a signed graph.

Definition 4.5.13. *The $i-$ stability, denoted as $\gamma_s^+(\Sigma)$, is the minimum number of vertices that must be removed to increase the domination number in Σ.*

Definition 4.5.14. *The $d-$ stability, denoted as $\gamma_s^-(\Sigma)$, is the minimum number of vertices that must be removed to reduce the domination number in Σ.*

Following are some of the results.

Observation 4.5.15. *For a signed path Σ on n vertices where $n \equiv 0(mod\,4)$ whose edges are arranged in a positive and negative sign alternatively, $i-$ stability does not exist.*

We proved for sufficiently large n, $\gamma_s^+(\Sigma)$ and $\gamma_s^-(\Sigma)$ are constant for signed paths and signed cycles.

Theorem 4.5.16. *Consider a signed path $\Sigma = (v_1, v_2, ..., v_n)$ on n vertices, and the edges of Σ are assigned as $+$ or $-$, respectively. Then*

$$\gamma_s^-(\Sigma) = \begin{cases} 1 : \text{if } n \text{ is odd or } n \equiv 0(mod\,4), n > 8 \\ 1 : \text{if } n \equiv 2(mod\,4), \text{ with } \delta^+(\Sigma) = 0 \\ 2 : \text{if } n \text{ is even } n \geq 6 \text{ with } \delta^-(\Sigma) = 0 \end{cases}$$

Theorem 4.5.17. *Consider a signed path $\Sigma = (v_1, v_2, ..., v_n)$ on n vertices, and the edges of Σ are assigned as $+$ or $-$, respectively. Then*

$$\gamma_s^+(\Sigma) = \begin{cases} 1 : \text{if } n \text{ is odd} \\ 2 : \text{if } n \text{ is even } n \geq 6 \text{ with } \delta^-(\Sigma) = 0 \\ 3 : \text{if } n \equiv 0(mod\,4), n > 8 \text{ with } \delta^+(\Sigma) = 0 \\ 1 : \text{if } n \equiv 2(mod\,4), \text{ with } \delta^+(\Sigma) = 0 \end{cases}$$

Theorem 4.5.18. *Consider a signed cycle $\Sigma = \{v_1, v_2, ..., v_n\}$ on n vertices, and the edges of Σ are assigned as $+$ or $-$, respectively. Then*

$$\gamma_s^-(\Sigma) = \begin{cases} 1 : \textit{if } n \textit{ is odd with } \delta^+(\Sigma) = 0 \\ 1 : \textit{if } n \equiv 1(mod\,4) \textit{ with } \delta^-(\Sigma) = 0 \\ 3 : \textit{if } n \equiv 3(mod\,4), \textit{ with } \delta^-(\Sigma) = 0 \\ 2 : \textit{if } n \equiv 0(mod\,4), n \geq 12 \\ 1 : \textit{if } n \equiv 2(mod\,4), n \geq 14 \end{cases}$$

Definition 4.5.19. *A vertex $v \in V(\Sigma)$ is stable if $\gamma_s(\Sigma - v) = \gamma_s(\Sigma)$.*

Definition 4.5.20. *A signed graph Σ is γ_s-stable if $\gamma_s(\Sigma - v) = \gamma_s(\Sigma)$ for any vertex $v \in V(\Sigma)$. If so, then $V = V^0$.*

Definition 4.5.21. *A signed graph Σ is k-stable if it is stable and $\gamma_s(\Sigma) = k$.*

Observation 4.5.22. *If Σ has either an isolated vertex or negative pendant vertex, then Σ is not γ_s-stable signed graph. So we consider the signed graphs without isolated vertex and negative pendant vertex.*

Corollary 4.5.23. *A signed graph Σ is γ_s-stable graph if and only if for each vertex v either*
(a) v is in every γ_s-set and there exist a dominating set S in $\Sigma - v$ such that $|S| = \gamma_s(\Sigma)$ and $S \subseteq V(\Sigma) - N[v]$ or
(b) There is no $\gamma_s(\Sigma)$-set S such that $v \notin S$ and $S \cap N^+(v) = \{u\}$ for some vertex $u \in S$.

We characterized $\Sigma \cong cor(\Sigma^*)$ with respect to k-stable.

Theorem 4.5.24. *Let Σ be a connected signed graph of order n having at least one positive pendant vertex and no negative pendant vertex. Then Σ is k-stable signed graph if and only if $\Sigma \cong cor(\Sigma^*)$ for some connected signed graph Σ^* of order k, $\delta^-(\Sigma^*) \geq 1$.*

Lemma 4.5.25. *If v is a stable vertex of a signed tree Σ, $\gamma_s(\Sigma - v) = \gamma_s(\Sigma)$.*

We characterized the signed tree with respect to γ_s-critical.

Theorem 4.5.26. *A signed tree Σ of order $n \geq 3$ is γ_s-critical if and only if each vertex v of Σ is either a negative leaf or a support vertex or $d^-(v) \geq d^+(v)$.*

Problem 4.5.27. *Characterize the signed graphs for which $|V^+(\Sigma)| = |V^-(\Sigma)|$.*

Problem 4.5.28. *Find the bounds for $|E^+(\Sigma)|$ and $|E^-(\Sigma)|$.*

4.6 Apex of Fuzzy Vertices and Bottom of Fuzzy Vertices in a Fuzzy Graph

In **Chapter 6**, we studied some results on apex of P and bottom of P are given.

Definition 4.6.1. *A fuzzy graph $G = (V, \sigma, \mu)$ is a triple consisting of a nonempty set V together with a pair of functions $\sigma : V \to [0, 1]$ and $\mu : E \to [0, 1]$ such that for all $x, y \in V$, $\mu(xy) \leq \sigma(x) \wedge \sigma(y)$. The fuzzy set σ is called the fuzzy vertex set of G and μ the fuzzy edge set of G.*

Definition 4.6.2. *Let S be any nonempty set. A mapping $X : S \to [0, 1]$ is called a fuzzy subset of X.*

Definition 4.6.3. *Let S be a fuzzy subset in a set M, the strongest fuzzy relation on M, that is a fuzzy relation T with respect to S given by $T(x, y) = min\{S(x), S(y)\}$ for all x and y in M.*

Definition 4.6.4. *Let M be any nonempty set, N be any set and $f : N \to M \times M$ be any function. Then P is a fuzzy subset of M, T is a fuzzy relation on M with respect to P and Q is a fuzzy subset of N such that $Q(n) \leq T(x, y)_{n \in f^{-1}(x,y)}$. Then the ordered triple $F_g = (P, Q, f)$ is called a fuzzy graph $(f_g,)$ where the elements of P are called fuzzy points $(f_p s)$ or fuzzy vertices and the elements of Q are called fuzzy lines or fuzzy edges of the fuzzy graph F.*

Definition 4.6.5. *Let $F_g = (P, Q, f)$ be a f_g. Then the bottom of P is defined by $d(P) = min\{P(x) : x \in M\}$.*

Definition 4.6.6. *Let $F_g = (P, Q, f)$ be a f_g. Then the apex of P is defined by $h(P) = max\{P(x) : x \in M\}$.*

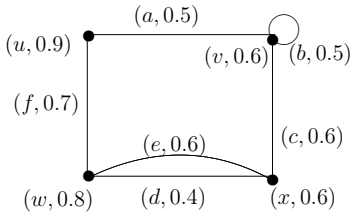

Figure 4.3: Fuzzy Graph.

For the fuzzy graph in Figure 4.3, $d(P) = 0.6$, $h(P) = 0.9$.

Here we observed the following.

Theorem 4.6.7. *Let $F_g = (P, Q, f)$ be any f_g with respect to set M and N where $|M| = v$ and $|N| = e$. Then $S(F_g) \leq eh(P)$.*

Theorem 4.6.8. *Let $F_g = (P, Q, f)$ be any f_{sig} $v-$fuzzy vertices. Then $\frac{2S(F_g)}{v(v-1)} \leq h(P)$.*

Theorem 4.6.9. *Let $F_g = (P, Q, f)$ be any f_{cg} with $v-$fuzzy vertices. Then $\frac{2S(F_g)}{v(v-1)} \leq h(P)$.*

Theorem 4.6.10. *Let $F_g = (P, Q, f)$ be a f_{cg} with $v-$fuzzy vertices and P be constant function if and only if $d(P) = \frac{2S(F_g)}{v(v-1)} = h(P)$.*

Theorem 4.6.11. *Let $F_g = (P, Q, f)$ be any f_g with respect to set M and N where $|M| = v$ and $|N| = e$. Then*

(i) $\frac{O(F_g)+S(F_g)}{v+e} \leq h(P)$

(ii) $\frac{O(F_g)-S(F_g)}{v-e} \leq h(P)$.

Theorem 4.6.12. *Let* $F_g = (P, Q, f)$ *be any* f_{sig} *with* $v-$ *fuzzy vertices. Then*

(i) $O(F_g) + S(F_g) \leq \frac{v(v+1)h(P)}{2}$

(ii) $O(F_g) - S(F_g) \leq \frac{v(3-v)h(P)}{2}$.

Theorem 4.6.13. *If* F_g *is a fuzzy* k_1- *regular graph with* $v-$ *fuzzy vertices with* $e \leq \frac{v(v-1)}{2}$, *then* $h(P) \geq \frac{k_1}{v-1}$.

Problem 4.6.14. *Study the domination parameters in fuzzy graphs.*

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LIST OF PUBLICATIONS

1. **P. Jeyalakshmi** and K. Karuppasamy, Connected Domination in a Signed graph and its Complement, *Journal of Intelligent and Fuzzy Systems* **45** (1) (2023), 345-356. (IF: 2)

2. **P. Jeyalakshmi** and K. Karuppasamy, Apex of fuzzy vertices and bottom of fuzzy vertices in a fuzzy graph, *Advances in Mathematics: Scientifific Journal* **9**(3) (2020), 979-989.

3. **P. Jeyalakshmi,** K. Karuppasamy and S.Arockiaraj , Independent Domination in Signed Graphs, *Discrete Mathematics, Algorithms and Applications* **13** (5) (2021), 2150065 -2150074.

4. **P. Jeyalakshmi** and K. Karuppasamy, Domination Changing and Unchanging Signed Graphs upon the Vertex Removal, *TWMS J. App. and Eng. Math.* (in press).